

One-loop Self Energies in Softly Broken Supersymmetric QED

Tino S. Nyawelo¹, Paolo Gondolo²,

*Department of Physics, University of Utah,
115 S 1400 E, # 201, Salt Lake City, UT 84112-0830, USA.*

Abstract

We apply the supergraph with spurion technique to compute the renormalization of one-loop diagrams that contributes to electron and selectron self energies in a softly broken Supersymmetric Quantum Electrodynamics (SQED). In particular, we calculate the one-loop gauge superfield contribution to the two-point function.

¹ E-mail: tnyawelo@physics.utah.edu

² E-mail: paolo@physics.utah.edu

1 Introduction

Supersymmetric theories have been always been famous for their extraordinary renormalization properties. One of them being the absence of quadratic divergences. It is this property that has led to the development of the Minimal Supersymmetric Standard Model (MSSM).

It is known for a long time that if supersymmetry is realized in nature it must be broken because no super multiplet has been observed. Models of spontaneously broken supersymmetry were constructed [1] and their renormalization was studied extensively [2, 3].

Gauge theories with softly broken supersymmetry have been widely studied. Almost all of the parameters of the Minimal Supersymmetric Standard Model (MSSM) encode possible ways in which supersymmetry is broken in a soft way, while preserving the gauge symmetries of the MSSM. To break supersymmetry without destroying the renormalization properties of globally supersymmetric theories, in particular the non-renormalization theorems and the cancellation of quadratic divergences, one has to introduce soft terms. The soft parameters have essentially been classified in by Girardello and Grisaru [4]. These authors gave a list of possible soft breaking term with the new logarithmic divergences they generate: They are either gaugino masses, Hermitian and complex mass matrices for the complex scalars or three-linear scalar couplings. These masses can either be introduced on the component level of the supersymmetric theory, or described by spurion insertions, θ^2 and $\bar{\theta}^2$. The latter approach is very powerful because it leaves most of the supersymmetric structure intact. In this paper, we describe supersymmetry breaking terms in a way that takes most advantage of the special properties of supersymmetric theories.

In previous publication [5] we investigated the effective action of (softly) broken supersymmetric theories at the one loop level. We focused on the renormalization of soft parameters in a softly broken supersymmetric models. Since our one loop expressions for Kähler potential and for the soft parameters are complicated, it is not easy to extract the renormalization information such as the wave function renormalization Z and masses renormalization Z_m . In this paper we perform supergraph computations of the one-loop diagrams that contributes to electron and selectron self energies in globally softly broken Supersymmetric Quantum Electrodynamics (SQED) from which one can read off these renormalization constants. We employ again the standard supergraph techniques that can be found in the textbooks [6, 7], and use spurions θ^2 and $\bar{\theta}^2$ to parameterize soft supersymmetry breaking terms. We find that our one-loop result generates divergent terms involving spurions θ^2 and $\bar{\theta}^2$ [4, 8, 9], which we removed by Yamada spurion dependent field redefinition [9]. We follow the convention³ of textbook by Wess and Bagger [10].

The plan of the paper is as follows: In sections 2 and 3 we introduce the classical action of the extended model of the SQED including the soft supersymmetry breaking and calculate the associated superpropagators. The computation of the one-loop diagrams that contributes to electron and selectron is performed in section 4. In subsection 4.1, we first consider the one loop self energy due to the soft selectron mass insertions, because it is easier than the ones that result from soft photino mass insertions. Its contribution is described in details in 4.2. In both cases we find that the divergent part of the wave function renormalization is independent of the soft breaking, and is the same for selectron and the electron. In section 5, we introduce counter terms that are needed to cancel divergences we

³Our conventions for Gamma matrices are slightly different from those of [10]: $\gamma^m = \begin{pmatrix} 0 & i\sigma^m \\ i\bar{\sigma}^m & 0 \end{pmatrix}$, and $\gamma_5 = \begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & -\mathbb{1}_2 \end{pmatrix}$.

encountered in these examples.

Finally, there are two appendices. Appendix A, gives some relations which we used in 4.2 to simplify spurion operators when they are sandwiched between two superspace delta functions $\delta_{21} = \delta^4(x_2 - x_1) \delta^4(\theta_2 - \theta_1)$, as naturally happens in the evaluation one loop supergraphs. Appendix B, describes the regularization by dimensional reduction of the divergent integrals we encountered in section 4.

2 Super Quantum Electrodynamics

In this section we consider globally Super Quantum Electrodynamics with soft supersymmetry breaking interactions. The theory of Super Quantum Electrodynamics consists of two oppositely charged chiral multiplets $\Phi_+ = (\varphi_+, \psi_+, F_+)$ and $\Phi_- = (\varphi_-, \psi_-, F_-)$ under a U(1) gauge symmetry of which $V = (A_m, \lambda^\alpha, \bar{\lambda}^{\dot{\alpha}}, \mathcal{D})$ is the vector superfield.

The components of these superfields for the vector multiplet V in the Wess–Zumino gauge and the chiral multiplets Φ_\pm are identified by (The vertical bar $|$ at the end of an expression indicates that we have set all $\theta_\alpha = \bar{\theta}_{\dot{\alpha}} = 0$.)

$$\begin{aligned} \varphi_\pm &= \Phi_\pm|, & \bar{\varphi}_\pm &= \bar{\Phi}_\pm|, & F_\pm &= \frac{D^2}{-4}\Phi_\pm|, & \bar{F}_\pm &= \frac{\bar{D}^2}{-4}\bar{\Phi}_\pm|, \\ \psi_\pm^\alpha &= \frac{D^\alpha}{\sqrt{2}}\Phi_\pm|, & \bar{\psi}_\pm^{\dot{\alpha}} &= \frac{\bar{D}^{\dot{\alpha}}}{\sqrt{2}}\bar{\Phi}_\pm|, & \lambda_\alpha &= -\frac{i}{4}D_\alpha\bar{D}^2V|, & \bar{\lambda}_{\dot{\alpha}} &= \frac{i}{4}\bar{D}_{\dot{\alpha}}D^2V|, \\ \sigma_{\alpha\dot{\alpha}}^m A_m &= -\frac{1}{2}[D_\alpha, \bar{D}_{\dot{\alpha}}]V|, & \bar{D}^2 D^2 V| &= 8(\mathcal{D} + i\partial_m A^m). \end{aligned} \quad (1)$$

Here (\mathcal{D}, F_\pm) are auxiliary scalar fields, φ_+ and φ_- are respectively left-handed and right-handed selectron fields⁴; and A_m is a real vector field, the photon. The complex Weyl spinors $(\bar{\psi}_\pm^{\dot{\alpha}}, \psi_\pm^\alpha)$ and $(\bar{\lambda}^{\dot{\alpha}}, \lambda^\alpha)$ are combined to form one massive Dirac spinor, the electron and a photino Majorana spinor respectively (see (7)).

The supersymmetric action for the SQED theory is given by

$$S_{\text{susy}} = \int d^8z \left(\bar{\Phi}_+ e^{+2eV} \Phi_+ + \bar{\Phi}_- e^{-2eV} \Phi_- \right) + \left\{ \int d^6z \left(m \Phi_+ \Phi_- + \frac{1}{4} W^\alpha W_\alpha \right) + \text{h.c.} \right\}, \quad (2)$$

where we use the full and chiral superspace measures, $d^8z = d^4x d^4\theta$ and $d^6z = d^4x d^2\theta$, respectively, m is the mass of the electron superfield and e is the U(1) charge. In this action we have introduced the Abelian superfield strength

$$W_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V, \quad (3)$$

with the components

$$\begin{aligned} W_\alpha| &= -i\lambda_\alpha, & D_\beta W_\alpha| &= -i(\sigma^{mn}\epsilon)_{\beta\alpha} F_{mn} - \epsilon_{\beta\alpha} \mathcal{D}, & D^2 W_\alpha| &= -4\sigma_{\alpha\dot{\alpha}}^m \partial_m \bar{\lambda}^{\dot{\alpha}}, \\ \bar{W}_{\dot{\alpha}}| &= i\bar{\lambda}_{\dot{\alpha}}, & \bar{D}_{\dot{\beta}} \bar{W}_{\dot{\alpha}}| &= -i(\epsilon \bar{\sigma}^{mn})_{\dot{\beta}\dot{\alpha}} F_{mn} - \epsilon_{\dot{\beta}\dot{\alpha}} \mathcal{D}, & \bar{D}^2 \bar{W}_{\dot{\alpha}}| &= -4\partial_m \lambda^\alpha \sigma_{\alpha\dot{\alpha}}^m. \end{aligned} \quad (4)$$

⁴The notations $\varphi_+ \equiv \varphi_L$, and $\varphi_- \equiv \varphi_R^*$ are current in the literature, but not always convenient in displaying equations.

A Fayet–Iliopoulos term can be included, but we have not done so here.

To include soft supersymmetry breaking interactions we extend this theory by including the following soft action

$$S_{\text{soft}} = \int d^6z \theta^2 \left(M^2 \Phi_+ \Phi_- + \frac{1}{2} m_{\tilde{\gamma}} W^\alpha W_\alpha \right) + \text{h.c.} , \quad (5)$$

where M is a complex scalar mass, and $m_{\tilde{\gamma}}$ is the photino mass. The factor in front of the photino mass $m_{\tilde{\gamma}}$ has been chosen such that the normalization of the kinetic term of the gaugino is taken into account.

The component form of the full SQED action, after eliminating the auxiliary fields (\mathcal{D}, F_\pm) and their complex conjugates reads

$$\begin{aligned} S_{\text{full}} &= S_{\text{susy}} + S_{\text{soft}} \\ &= \int d^4x \left[-\frac{1}{4} F_{mn} F^{mn} - \frac{1}{2} \tilde{\gamma} (\not{\partial} - \tilde{m}_{\tilde{\gamma}}) \tilde{\gamma} - \left| \partial_m \varphi_\pm \mp i e A_m \varphi_\pm \right|^2 - \bar{\Psi}_D (\not{\partial} - i e \not{A} + m) \Psi_D \right. \\ &\quad - m^2 (\bar{\varphi}_+ \varphi_+ + \bar{\varphi}_- \varphi_-) + M^2 (\varphi_+ \varphi_- + \bar{\varphi}_+ \bar{\varphi}_-) - \frac{1}{2} e^2 (\bar{\varphi}_+ \varphi_+ - \bar{\varphi}_- \varphi_-)^2 \\ &\quad \left. - i \sqrt{2} e \left(\varphi_+ \bar{\Psi}_D P_R \tilde{\gamma} + \bar{\varphi}_+ \tilde{\gamma} P_L \Psi_D - \bar{\varphi}_- \tilde{\gamma} P_R \Psi_D - \varphi_- \bar{\Psi}_D P_R \tilde{\gamma} \right) \right] , \end{aligned} \quad (6)$$

where the notation \pm indicates that we sum over $+$ and $-$. In this expression, we have introduced the electron Dirac spinor and the photino Majorana spinor

$$\Psi_D = \begin{pmatrix} \psi_{+\alpha} \\ \bar{\psi}_{-\dot{\alpha}} \end{pmatrix} = \Psi_{+L} + \Psi_{-R} , \quad \tilde{\gamma} = \begin{pmatrix} \lambda_\alpha \\ \bar{\lambda}_{\dot{\alpha}} \end{pmatrix} , \quad P_L = \frac{1}{2} (\mathbb{1}_4 + \gamma_5) , \quad P_R = \frac{1}{2} (\mathbb{1}_4 - \gamma_5) . \quad (7)$$

3 The Superpropagators

After the strictly classical discussion we now turn towards the quantization of the theory using path integral methods. To this end we need to determine the propagators of the superfields Φ_+ , Φ_- , V by coupling them to the sources J_+ , J_- , J_V respectively. Because of the super gauge invariance the kinetic operator of the vector multiplet is not invertible. This requires gauge fixing and the introduction of the corresponding supersymmetric Fadeev–Popov ghosts C, C', \bar{C}, \bar{C}' (see e.g. [6, 7]). Since the gauge superfields appear in the same way as in the supersymmetry preserving theory, we use the gauge fixing action [11] (see also [12])

$$S_{\text{G.F.}} = - \int d^8z \Theta \bar{\Theta} , \quad \Theta = \sqrt{2} \frac{\bar{D}^2}{-4} V \quad (8)$$

as if supersymmetry is unbroken. This implies that the FP–ghost sector is the same as in the supersymmetric theory in the Feynman–’t Hooft gauge [11, 12].

By using the vector notation

$$\Phi = \begin{pmatrix} \Phi_+ \\ \Phi_- \end{pmatrix} , \quad \bar{\Phi} = (\bar{\Phi}_+ \quad \bar{\Phi}_-) , \quad J = \begin{pmatrix} J_+ \\ J_- \end{pmatrix} , \quad \bar{J} = (\bar{J}_+ \quad \bar{J}_-) , \quad (9)$$

for the electron superfields and the chiral sources, we can write the quadratic chiral and vector superfields action after gauge fixing as

$$S_{\text{quad}} = \int d^8z \left\{ \bar{\Phi} P_+ \Phi + \left(\frac{1}{2} \Phi^T \left[m P_- + M^2 \eta_- \right] \frac{D^2}{-4\Box} \Phi + J^T \frac{D^2}{-4\Box} \Phi + \text{h.c.} \right) \right. \\ \left. - V \left(\Box - m_{\tilde{\gamma}} \Box^{1/2} \eta_V - m_{\tilde{\gamma}} \Box^{1/2} \bar{\eta}_V - J_V \right) V \right\}. \quad (10)$$

Here we have made use of the chiral projection operators

$$P_+ = \frac{\bar{D}^2 D^2}{16\Box}, \quad P_- = \frac{D^2 \bar{D}^2}{16\Box}, \quad (11)$$

and spurion operators

$$\eta_{\pm} = P_{\pm} \theta^2 P_{\pm}, \quad \bar{\eta}_{\pm} = P_{\pm} \bar{\theta}^2 P_{\pm}, \quad \eta_V = \frac{D^{\alpha} \theta^2 \bar{D}^2 D_{\alpha}}{8\Box^{1/2}}, \quad \bar{\eta}_V = \frac{\bar{D}_{\dot{\alpha}} \bar{\theta}^2 D^2 \bar{D}^{\dot{\alpha}}}{8\Box^{1/2}}. \quad (12)$$

Furthermore, we have used the fact that $P_+ \Phi = \Phi$, $P_- \bar{\Phi} = \bar{\Phi}$ to rewrite the integral as a full superspace integral. From the quadratic superfield actions (10) we determine the propagators for chiral and vector superfields.

We begin with the chiral multiplet. The propagators of chiral multiplet are obtained from the action (10) by rewriting the quadratic action for the chiral superfields as

$$S_{\Phi} = \int d^8z \left\{ \frac{1}{2} (\Psi^T \quad \bar{\Psi}) \Delta^{-1} \begin{pmatrix} \Psi \\ \bar{\Psi}^T \end{pmatrix} + (\bar{J} \quad J^T) \begin{pmatrix} \Psi \\ \bar{\Psi}^T \end{pmatrix} \right\}, \quad (13)$$

using the field redefinitions

$$\Phi^T = \frac{\bar{D}^2}{-4} \bar{\Psi}, \quad \bar{\Phi}^T = \frac{D^2}{-4} \Psi. \quad (14)$$

Here the superscripts T denotes transposition, $\Psi(\bar{\Psi})$ is (anti-)chiral, and the quadratic operator Δ^{-1} is given by

$$\Delta^{-1} = \begin{pmatrix} (m + M^2 \bar{\theta}^2) \frac{D^2}{-4} & \Box P_- \\ \Box P_+ & (m + M^2 \theta^2) \frac{\bar{D}^2}{-4} \end{pmatrix}. \quad (15)$$

The path integral can be evaluated in the usual way and we find that the functional integral

$$Z_0(J, \bar{J}) = \int \mathcal{D}\Phi \mathcal{D}\bar{\Phi} e^{i S_{\Phi}} = \exp \left\{ - \frac{i}{2} \int d^8z (\bar{J} \quad J^T) \Delta \begin{pmatrix} \bar{J}^T \\ J \end{pmatrix} \right\}. \quad (16)$$

To proceed we must invert Δ^{-1} . We note that the quadratic operator Δ^{-1} can be decomposed into a standard free part P and perturbation L

$$\Delta^{-1} = P^{-1} + L, \quad P^{-1} = \Box \begin{pmatrix} 0 & P_- \\ P_+ & 0 \end{pmatrix}, \quad L = \begin{pmatrix} (m + M^2 \bar{\theta}^2) \frac{D^2}{-4} & 0 \\ 0 & (m + M^2 \theta^2) \frac{\bar{D}^2}{-4} \end{pmatrix}. \quad (17)$$

After a long and tedious computation, the inverse of Δ^{-1} can be cast into the form:

$$\Delta = P \left(\mathbb{1} - (LP)^2 \right)^{-1} \left(\mathbb{1} - LP \right) = \frac{1}{\square - m^2} \begin{pmatrix} \overline{C} & P_+ \\ P_- & C \end{pmatrix} + \frac{M^2}{(\square - m^2)^2 - M^4} \begin{pmatrix} A_+ \overline{C} & A_+ \\ A_- & A_- C \end{pmatrix}, \quad (18)$$

where the matrices A_+ , A_- , C and \overline{C} are given by

$$\begin{aligned} A_+ &= m \eta_+ + m \bar{\eta}_+ + \frac{M^2}{\square - m^2} \left(\square \eta_+ \bar{\eta}_+ + m^2 \bar{\eta}_+ \eta_+ \right), & C &= \left(m + M^2 \bar{\theta} \right) \frac{D^2}{4 \square}, \\ A_- &= m \eta_- + m \bar{\eta}_- + \frac{M^2}{\square - m^2} \left(\square \bar{\eta}_- \eta_- + m^2 \bar{\eta}_- \eta_- \right), & \overline{C} &= \left(m + M^2 \theta^2 \right) \frac{\overline{D}^2}{4 \square}. \end{aligned} \quad (19)$$

Notice that $A_- = A_+^T$ and that $\overline{C} = C^\dagger$. Inserting (18) into (16) we obtain the full propagator with the spurion supersymmetry breaking [13]

$$-\frac{i}{2} \int d^8 z \begin{pmatrix} \overline{J} & J^T \end{pmatrix} \Delta \begin{pmatrix} \overline{J}^T \\ J \end{pmatrix} = -\frac{i}{2} \int d^8 z \left(\overline{J}_\pm \Delta_{\Phi_\pm \Phi_\pm} J_\pm - J_\pm \Delta_{\Phi_\pm \Phi_\pm} J_\pm + \text{h.c.} \right). \quad (20)$$

The notation \pm indicates that we sum over $+$ and $-$. From this expression, we can read off the propagators:

$$\begin{aligned} \Delta_{\Phi_\pm \Phi_\pm} &= \frac{1}{\square - m^2} + \frac{M^2}{(\square - m^2)^2 - M^4} \left\{ m \theta^2 + m \bar{\theta}^2 + \frac{M^2}{\square - m^2} \left(\frac{\overline{D}^2}{-4} \theta^2 \bar{\theta}^2 \frac{D^2}{-4} + m^2 \theta^2 \bar{\theta}^2 \right) \right\} \\ \Delta_{\Phi_\pm \Phi_\pm} &= \frac{m}{\square - m^2} \frac{D^2}{-4 \square} + \frac{M^2}{(\square - m^2)^2 - M^4} \left\{ \frac{D^2}{-4} \bar{\theta}^2 + m^2 \frac{D^2}{-4 \square} \bar{\theta}^2 \right. \\ &\quad \left. + \frac{m M^2}{\square - m^2} \bar{\theta}^2 \left(\frac{D^2}{-4} \theta^2 + \theta^2 \frac{D^2}{-4} \right) \right\}. \end{aligned} \quad (21)$$

For the vector multiplets we can perform a very similar analysis to compute the inverse of the kinetic operator Δ_V . Writting the quadratic vector superfield action from (10) in terms of projection operator $P_V = 1 - \mathbb{1}_V$ we find that

$$S_V = - \int d^8 z V \left\{ \Delta_V^{-1} - J_V \right\} V, \quad (22)$$

with the quadratic operator Δ_V^{-1} and the identity matrix $\mathbb{1}_V$ given by

$$\Delta_V^{-1} = \square P_V + \left(\square \mathbb{1}_V - m_{\tilde{\gamma}} \square^{1/2} \eta_V - m_{\tilde{\gamma}} \square^{1/2} \bar{\eta}_V \right), \quad \mathbb{1}_V = \eta_V \bar{\eta}_V + \bar{\eta}_V \eta_V. \quad (23)$$

The path integral can be carried out and gives

$$Z_V(J) = \int \mathcal{D}V e^{i S_V} = \exp \left\{ \frac{i}{4} \int d^8 z J_V \Delta_V J_V \right\}. \quad (24)$$

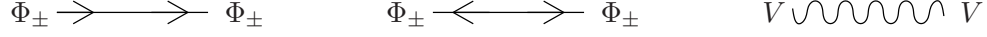


Figure 1: This picture gives our drawing conventions for the propagators which we employ throughout this paper. The first two diagrams correspond to the chiral propagators defined in (21): The first one represents $\Delta_{\bar{\Phi}_{\pm}\Phi_{\pm}}$ and the second one $\Delta_{\Phi_{\pm}\Phi_{\pm}}$. The latter refer to the vector propagator given in (25).

The vectorfield superpropagator can be read off directly from (24) and reads⁵

$$\begin{aligned}\Delta_V &= \frac{1}{\square} + \frac{m_{\tilde{\gamma}}^2}{\square(\square - m_{\tilde{\gamma}}^2)} \mathbb{1}_V + \frac{m_{\tilde{\gamma}}}{\square(\square - m_{\tilde{\gamma}}^2)} (\square^{1/2} \eta_V + \square^{1/2} \bar{\eta}_V) \\ &= \frac{1}{\square} + \left\{ \frac{m_{\tilde{\gamma}}}{\square(\square - m_{\tilde{\gamma}}^2)} \left(\frac{m_{\tilde{\gamma}}}{32\square} D^{\alpha} \bar{D}^2 \theta^2 \bar{\theta}^2 D^2 \bar{D}^{\dot{\alpha}} \sigma_{\alpha\dot{\alpha}}^m \partial_m + \frac{1}{8} D^{\alpha} \bar{D}^2 \theta^2 D_{\alpha} \right) + \text{h.c.} \right\} \quad (25)\end{aligned}$$

In figure 1 we have collected our graphical representation for these propagators.

4 Examples

As a quick application of our result, in this section we compute the one-loop chiral multiplet self-energy depicted in figure 2, in the presence of soft breaking terms. The classical action and the propagators were given in section 2 and 3. Here we write the vertices, after that we evaluate the Feynman graphs that lead to corrections of the gauge superfield contribution to the two-point $\Phi\bar{\Phi}$ -vertices. These contributions come from the following part of S_{susy}

$$S_{\text{susy}} \supset \int d^8z \left(2e \bar{\Phi} T V \Phi + 2e^2 \bar{\Phi} V^2 \Phi \right). \quad (26)$$

Here we have introduced the charge operator for the electron T

$$T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (27)$$

4.1 Self Energy due to Selectron Mass

We present examples of calculations of self-energy supergraph depicted in figure 2.A. For simplicity, the soft breaking term S_{soft} is restricted to selectron mass term M .

The relevant $2e \bar{\Phi} V \Phi$ interaction term is given in (26), and corresponding propagators are given by (21) and (25), but with $m_{\tilde{\gamma}} = 0$. To calculate this self energy graph the Φ , $\bar{\Phi}$ and V superfields are replaced by the corresponding functional derivatives with respect to sources that act on the exponential of the propagators (20) and (24). After functional derivations, the expression for the supergraph figure 2.a reads

$$i\Gamma_A(m_{\tilde{\gamma}} = 0) = -e^2 \int (d^8z)_{1234} \bar{\Phi}_2 \Phi_1 \frac{D_3^2}{-4} \delta_{31} (\Delta_{\Phi\bar{\Phi}})_3 \frac{\bar{D}_3^2}{-4} \delta_{32} \delta_{41} (\Delta_V)_4 \delta_{42}, \quad (28)$$

⁵We take this opportunity to point out that the term $\frac{1}{h\square - m_V^2}(1 - \mathbb{1}_v)$ is missing in the expression for the vector propagator eq.(58) in [5].

where $\Delta_{\bar{\Phi}\Phi}$ is given by (20), and $\Delta_V = \frac{1}{\square}$. Because (except for J_V) all these sources are chiral, the functional differentiation w.r.t. them leads to chiral delta function in superspace: $\frac{\delta J_{\pm 2}}{\delta J_{\pm 1}} = -\frac{1}{4}\bar{D}^2\delta_{12}$. Integrating over $z_3 = (x, \theta)_3$ and $z_4 = (x, \theta)_4$

$$\int d^8 z_3 F(z_3) \delta_{31} = F(z_1), \quad \int d^8 z_4 F(z_4) \delta_{41} = F(z_1), \quad (29)$$

we are left with

$$\begin{aligned} i\Gamma_A(m_{\tilde{\gamma}} = 0) = & -e^2 \int (d^8 z)_{12} \bar{\Phi}_2 \Phi_1 \left\{ \frac{1}{\square - m^2} \frac{D^2 \bar{D}^2}{16} + \frac{M^2}{(\square - m^2)^2 - M^4} \left[\left(m \frac{D^2}{-4} \theta^2 \frac{\bar{D}^2}{-4} + \text{h.c.} \right) \right. \right. \\ & \left. \left. + \frac{M^2}{\square - m^2} \left(m^2 \frac{D^2}{-4} \theta^2 \bar{\theta}^2 \frac{\bar{D}^2}{-4} + M^2 \frac{D^2}{-4} \frac{\bar{D}^2}{16} \theta^2 \bar{\theta}^2 \frac{D^2}{-4} \frac{\bar{D}^2}{-4} \right) \right] \right\}_1 \delta_{12} \frac{1}{\square_1} \delta_{12}. \end{aligned} \quad (30)$$

Using the relations

$$D^2 \bar{D}^2 D^2 = 16 \square D^2, \quad D^2 \theta^2 = -4 + 4 \theta^\alpha D_\alpha + \theta^2 D^2, \quad \bar{D}^2 \bar{\theta}^2 = -4 - 4 \bar{\theta}^{\dot{\alpha}} \bar{D}_{\dot{\alpha}} + \bar{\theta}^2 \bar{D}^2, \quad (31)$$

and the fact that supergraphs are only non-vanishing if they involve an equal number of super covariant derivatives D and \bar{D} , and of each of them at least two

$$\int (d^8 z)_{12} A_2 \delta_{12} [B D^2 \bar{D}^2]_2 \delta_{12} = 16 \int (d^8 z)_{12} A_2 \delta_{12} B_2 \delta_{12}, \quad (32)$$

we find that the supergraph, figure 2.A, becomes the following scalar integral

$$\begin{aligned} i\Gamma_A(m_{\tilde{\gamma}} = 0) = & -e^2 \int (d^8 z)_{12} \bar{\Phi}_2 \Phi_1 \left\{ \left[\frac{\square - m^2}{(\square - m^2)^2 - M^4} + \frac{M^2}{(\square - m^2)^2 - M^4} \left(m \theta^2 + m \bar{\theta}^2 \right. \right. \right. \\ & \left. \left. + \frac{M^2}{\square - m^2} \left[\theta^2 \bar{\theta}^2 (\square + m^2) + 2i \bar{\theta}^{\dot{\alpha}} \theta^\alpha \sigma_{\alpha\dot{\alpha}}^m \partial_m \right] \right) \right] \right\}_1 \delta_{12} \frac{1}{\square_1} \delta_{12}. \end{aligned} \quad (33)$$

As the integral over momentum space is symmetric under $p \rightarrow -p$, this means that after going to momentum space, the last term of (33) will not contribute, and hence we will set to zero now. The final step in evaluation of this diagram in the coordinate space representation is to make the expression local in the Grassmann variables. By integrating over θ_2

$$\int (d^8 z)_{12} A_2 \delta_{12} B_2 \delta_{12} = \int (d^4 x)_{12} d^4 \theta_1 A_2 \delta_{12}^4 B_2 \delta_{12}^4, \quad (34)$$

we find the following expression for (33)

$$\begin{aligned} i\Gamma_A(m_{\tilde{\gamma}} = 0) = & -e^2 \int (d^4 x)_{12} d^4 \theta \bar{\Phi}(x_2, \bar{\theta}) \Phi(x_1, \theta) \left[\frac{\square - m^2}{(\square - m^2)^2 - M^4} + \frac{M^2}{(\square - m^2)^2 - M^4} \left(m \theta^2 \right. \right. \\ & \left. \left. + m \bar{\theta}^2 + M^2 \left[\theta^2 \bar{\theta}^2 \frac{\square + m^2}{\square - m^2} \right] \right) \right] \delta_{12}^4 \frac{1}{\square_1} \delta_{12}^4. \end{aligned} \quad (35)$$

Since the expression only contains θ_1 , it is local in θ_1 and we simply dropped the subscript "1" on θ . In this expression $\delta_{21}^4 = \delta^4(x_2 - x_1)$ denotes the four dimensional space time delta function, and

the subscript 1 on the square bracket [...] denote that the corresponding expression is defined in superspace coordinate system 1. The final result in momentum space reads

$$i\Gamma_A(m_{\tilde{\gamma}} = 0) = \int \frac{d^4 p}{(4\pi)^4} \int d^4 \theta \left[\bar{\Phi}(p, \bar{\theta}) \Sigma(p; \theta, \bar{\theta}) \Phi(-p, \theta) \right], \quad (36)$$

from which the electron superfields self-energy $\Sigma(p; \theta, \bar{\theta})$ can be read off

$$\begin{aligned} \Sigma(p; \theta, \bar{\theta}) = & -e^2 \int \frac{d^4 q}{(2\pi)^4} \frac{1}{[(q + p/2)^2 + m^2]^2 - M^4} \frac{1}{(q - p/2)^2} \left[(q + p/2)^2 + m^2 \right. \\ & \left. - M^2 m(\theta^2 + \bar{\theta}^2) - \theta^2 \bar{\theta}^2 \left(M^4 - 2 \frac{m^2 M^4}{(q + p/2)^2 + m^2} \right) \right]. \end{aligned} \quad (37)$$

This integral (37) is divergent and therefore need to be regularized.

The component form of the diagram (35) before eliminating the auxiliary fields F_{\pm} and their complex conjugates \bar{F}_{\pm} reads

$$\begin{aligned} i\Gamma_A(m_{\tilde{\gamma}} = 0) = & -e^2 \int d^4 x_{12} \left\{ \left(\bar{\varphi}_{\pm}(x_2) \square \varphi_{\pm}(x_1) - i \bar{\psi}_{\pm}(x_2) \bar{\sigma}^m \partial_m \psi_{\pm}^{\alpha}(x_1) + \bar{F}_{\pm}(x_2) F_{\pm}(x_1) \right) \times \right. \\ & \times \left[\frac{\square - m^2}{(\square - m^2)^2 - M^4} \right]_1 + \left(\bar{F}_{\pm}(x_2) \varphi_{\pm}(x_1) + \bar{\varphi}_{\pm}(x_2) F_{\pm}(x_1) \right) \left[\frac{m M^2}{(\square - m^2)^2 - M^4} \right]_1 \\ & \left. + \bar{\varphi}_{\pm}(x_2) \varphi_{\pm}(x_1) \left[\frac{\square + m^2}{\square - m^2} \frac{M^4}{(\square - m^2)^2 - M^4} \right]_1 \right\} \delta_{12}^4 \frac{1}{\square_1} \delta_{12}^4. \end{aligned} \quad (38)$$

As it stands this expression (38) is logarithmically divergent and requires regularization. We have chosen to use dimensional reduction [14, 15]. In appendix B we have collected the one loop integrals calculated in this scheme. Using the standard scalar integrals J_2 and J_3 defined in appendix B (see B.12), we obtain

$$\begin{aligned} \Gamma_A(m_{\tilde{\gamma}} = 0) = & -\frac{1}{2} e^2 \int \frac{d^4 p}{(4\pi)^4} \left\{ \left(\bar{F}_{\pm}(p) F_{\pm}(-p) - \bar{\varphi}_{\pm}(p) p^2 \varphi_{\pm}(-p) - i \bar{\Psi}_D(p) \not{p} \Psi_D(-p) \right) \int_0^1 dx \left[J_2(\tilde{m}_-^2) \right. \right. \\ & \left. \left. + J_2(\tilde{m}_+^2) \right] + \left(\bar{F}_{\pm}(p) \varphi_{\pm}(-p) + \bar{\varphi}_{\pm}(p) F_{\pm}(-p) + M^2 \bar{\varphi}_{\pm}(p) \varphi_{\pm}(-p) \right) \int_0^1 dx \left[J_2(\tilde{m}_-^2) \right. \right. \\ & \left. \left. - J_2(\tilde{m}_+^2) \right] + 4 m^2 M^2 \bar{\varphi}_{\pm}(p) \varphi_{\pm}(-p) \int_0^1 dx \int_0^{1-x} dy \left[J_3(\tilde{M}_-^2) - J_3(\tilde{M}_+^2) \right] \right\}. \end{aligned} \quad (39)$$

In this expression we have combined the two Weyl spinors ψ_{\pm} into a charged Dirac spinor (7), the electron. The masses \tilde{m}_{\pm} and \tilde{M}_{\pm} are defined by

$$\tilde{m}_{\pm}^2 = x(1-x)p^2 + x m_{\pm}^2, \quad \tilde{M}_{\pm}^2 = m_{\pm}^2 y + m^2(1-x-y) + x(1-x)p^2, \quad (40)$$

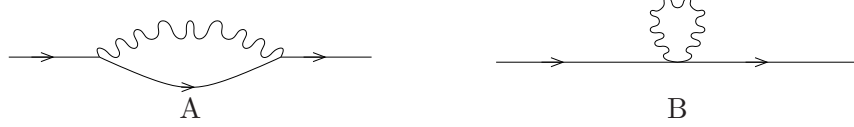


Figure 2: One-loop diagrams contributing to electron and selectron self energies.

with $m_{\pm}^2 = m^2 \pm M^2$. From the one-loop result given in (39) the wave function renormalization for the electron and its superpartner the selectron can be obtained from the divergent integral:

$$\frac{e^2}{2} \int_0^1 dx \left[J_2(\tilde{m}_-^2) + J_2(\tilde{m}_+^2) \right] = \frac{e^2}{16\pi^2} \frac{1}{\bar{\epsilon}} + \frac{e^2}{16\pi^2} \left[-2 + \ln \frac{m_{\pm}^2}{\mu^2} + \frac{p^2 + m_{\pm}^2}{p^2} \ln \frac{p^2 + m_{\pm}^2}{m_{\pm}^2} \right]. \quad (41)$$

where $\frac{1}{\bar{\epsilon}} = 1/\epsilon - \gamma_E + \ln(4\pi)$. From this expression, we can read off the wave function renormalization for the electron [16] and selectron

$$Z_1 = Z_{\bar{\varphi}_{\pm}\varphi_{\pm}} = Z_{\bar{\Psi}_D\Psi_D} = \mu^{-2\epsilon} \left(1 - \frac{e^2}{16\pi^2} \frac{1}{\bar{\epsilon}} \right). \quad (42)$$

As for the divergences arising from the integrals $\int_0^1 dx [J_2(\tilde{m}_-^2) - J_2(\tilde{m}_+^2)]$ cancel each other, and hence the second and third line of (39) are finite renormalizations for the selectron soft mass M .

This computation confirm that the divergent part of the wave function renormalization is independent of the soft breaking, and is the same for selectron and the electron. A consistency check on this result is obtained when one considers the well-known one beta functions [17–21] (for results including two loop beta functions see [9, 22]) for the parameters M and m , if we restrict to renormalizable models. In our previous paper [5], we obtain these beta functions by computing the renormalization of the parameters M and m , and found exact agreement. The absence of one-loop corrections to the electron mass m is a result of the well known non-renormalization theorem for the superpotential $\int d^6z (m \Phi_+ \Phi_- + \text{h.c.})$ [23] (see also [24–26]). Due to the non-renormalization, one also obtained the electron mass m is renormalized via the wave function renormalization Z_1 like

$$m = Z_1^{-1} m_R, \quad (43)$$

where m_R is the renormalized mass (For details see the textbook [27]).

4.2 Self Energy due to Photino Mass

We compute the one-loop self energy corrections due to the soft photino mass insertions depicted 2. We first consider the supergraph figure 2.A. After functional derivations, the expression for the supergraph figure reads

$$i\Gamma_A(M=0) = -e^2 \int (d^8z)_{12} \bar{\Phi}_2 \Phi_1 \left[\Delta_{\Phi\Phi} \frac{D^2 \bar{D}^2}{16} \right]_1 \delta_{12} (\Delta_V)_1 \delta_{12}, \quad (44)$$

where $\Delta_{\bar{\Phi}\Phi}$ and Δ_V are respectively given by (20) and (25), but with selectron mass $M = 0$. To reduce this integral to a scalar integral we partially integrate the $\bar{D}^2 D^2$ that acts on the first δ_{21} , to obtain

$$\begin{aligned} i\Gamma_A(M=0) = & -\frac{e^2}{16} \int d^8 z_{12} \bar{\Phi}_2 \left\{ \Phi_1 (\Delta_{\bar{\Phi}\Phi})_1 \delta_{12} [\bar{D}^2 D^2]_1 + (\bar{D}^2 D^2 \Phi)_1 (\Delta_{\bar{\Phi}\Phi})_1 \delta_{12} \right. \\ & + 2(\bar{D}_{\dot{\alpha}} D^2 \Phi)_1 (\Delta_{\bar{\Phi}\Phi})_1 \delta_{12} \bar{D}_1^{\dot{\alpha}} - 2(D_{\alpha} \Phi)_1 (\Delta_{\bar{\Phi}\Phi})_1 \delta_{12} [\bar{D}^2 D^{\alpha}]_1 \\ & \left. - 4(\bar{D}_{\dot{\alpha}} D_{\alpha} \Phi)_1 (\Delta_{\bar{\Phi}\Phi})_1 \delta_{12} [\bar{D}^{\dot{\alpha}} D^{\alpha}]_1 + (D^2 \Phi)_1 (\Delta_{\bar{\Phi}\Phi})_1 \delta_{12} \bar{D}_1^2 \right\} (\Delta_V)_1 \delta_{12} . \quad (45) \end{aligned}$$

Observe here that after partial integrating the $\bar{D}^2 D^2$ that acts on the first δ_{21} , the spurion operators $\mathbb{1}_V$ and η_V (which are hidden in vector superpropagator Δ_V) find themselves surrounded by two superspace delta functions δ_{21} . By integrating over the full double superspace, we obtained the identities given in appendix A. Using these identities (A.1–A.7), the last term of (45) vanishes, and we find that remaining terms becomes the following scalar integral

$$\begin{aligned} i\Gamma_A(M=0) = & -e^2 \int d^4 x_{12} d^4 \theta \bar{\Phi}_2 \left\{ \Phi_1 \frac{1}{(\square - m^2)_1} \delta_{12}^4 \frac{1}{\square_1} - m_{\tilde{\gamma}}^2 D_{\beta} \Phi \theta^{\beta} \frac{1}{(\square - m^2)_1} \delta_{12}^4 \left[\frac{1}{\square(\square - m_{\tilde{\gamma}}^2)} \right]_1 \right. \\ & + m_{\tilde{\gamma}} \bar{D}_{\dot{\beta}} D^2 \Phi_1 \left(\frac{1}{4} \bar{\theta}^{\dot{\beta}} - \frac{1}{2} \theta^2 \bar{\theta}^{\dot{\beta}} m_{\tilde{\gamma}} \right) \frac{1}{(\square - m^2)_1} \delta_{12}^4 \left[\frac{1}{\square(\square - m_{\tilde{\gamma}}^2)} \right]_1 \\ & - m_{\tilde{\gamma}}^2 (\bar{D}_{\dot{\beta}} D_{\beta} \Phi)_1 \frac{1}{(\square - m^2)_1} \delta_{12}^4 \left[\bar{\theta}^{\dot{\beta}} \theta^{\beta} \frac{1}{\square(\square - m_{\tilde{\gamma}}^2)} + \bar{\theta}^{\dot{\alpha}} \theta^{\gamma} \epsilon^{\beta\alpha} \epsilon^{\dot{\beta}\dot{\gamma}} \sigma_{\dot{\alpha}\alpha}^m \sigma_{\dot{\gamma}\gamma}^n \frac{\partial_m \partial_n}{\square^2(\square - m_{\tilde{\gamma}}^2)} \right]_1 \\ & \left. + 2 m_{\tilde{\gamma}} (\square \Phi)_1 \left(\theta^2 + \bar{\theta}^2 - 2 \bar{\theta}^2 \theta^2 m_{\tilde{\gamma}} \right) \frac{1}{(\square - m^2)_1} \delta_{12}^4 \left[\frac{1}{\square(\square - m_{\tilde{\gamma}}^2)} \right]_1 \right\} \delta_{12}^4 , \quad (46) \end{aligned}$$

where we have discarded the terms containing ∂_m because they are antisymmetric under $\partial_m \rightarrow \partial_m$.

Next we turn to supergraph figure 2.B. The two-point vertex $2e^2 \bar{\Phi} V^2 \Phi$ that give rise to that supergraph is obtained from (26). Using standard supergraphs techniques we find that the supergraph figure 2.B, becomes the following scalar integral

$$i\Gamma_B = e^2 \int (d^8 z)_{12} \bar{\Phi}_1 \Phi_1 \delta_{21} (\Delta_V)_2 \delta_{21} , \quad (47)$$

with the vector superfield propagator given in (25). Upon using A.1 in appendix A the supergraph (47) becomes the following scalar integral

$$i\Gamma_B = e^2 \int d^4 x_{12} d^4 \theta \bar{\Phi}_1 \Phi_1 \delta_{21}^4 \left[2 \frac{m_{\tilde{\gamma}}}{\square(\square - m_{\tilde{\gamma}}^2)} (\theta^2 + \bar{\theta}^2) - 4 \frac{m_{\tilde{\gamma}}^2}{\square(\square - m_{\tilde{\gamma}}^2)} \theta^2 \bar{\theta}^2 \right]_2 \delta_{21}^4 . \quad (48)$$

By combining these results (46) and (48), and computing the component action, we find

$$\begin{aligned}
i\Gamma_{\text{full}} &= i\Gamma_A(M=0) + i\Gamma_B \\
&= -e^2 \int d^4x_{12} \left\{ \left(\bar{F}_{\pm}(x_2)F_{\pm}(x_1) + \bar{\varphi}_{\pm}(x_2)\square\varphi_{\pm}(x_1) - i\psi_{\pm}(x_1)\sigma^m\partial_m\bar{\psi}_{\pm}(x_2) \right) \times \right. \\
&\quad \times \frac{1}{(\square - m^2)_1} \delta_{12}^4 \frac{1}{\square_1} \delta_{12}^4 + m_{\tilde{\gamma}}^2 \left(2\bar{F}_{\pm}(x_2)F_{\pm}(x_1) + 2\bar{\varphi}_{\pm}(x_2)\square\varphi_{\pm}(x_1) \right. \\
&\quad \left. \left. + i5\bar{\psi}_{\pm}(x_2)\bar{\sigma}^m\partial_m\psi_{\pm}(x_1) \right) \frac{1}{(\square - m^2)_1} \delta_{12}^4 \left[\frac{1}{\square(\square - m_{\tilde{\gamma}}^2)} \right]_1 \delta_{12}^4 - m_{\tilde{\gamma}}^2 \left(2\bar{\varphi}_{\pm}(x_2)\partial^m\partial^n\varphi_{\pm}(x_1) \right. \\
&\quad \left. \left. + i2\bar{\psi}_{\pm}(x_2)\bar{\sigma}^m\partial^n\psi_{\pm}(x_1) + n \leftrightarrow m \right) \frac{1}{(\square - m^2)_1} \delta_{12}^4 \left[\frac{\partial_m\partial_n}{\square^2(\square - m_{\tilde{\gamma}}^2)} \right]_1 \delta_{12}^4 \right. \\
&\quad \left. - 2m_{\tilde{\gamma}} \left(\bar{F}_{\pm}(x_1)\varphi_{\pm}(x_1) + \bar{\varphi}_{\pm}(x_1)F_{\pm}(x_1) - 2m_{\tilde{\gamma}}\bar{\varphi}_{\pm}(x_1)\varphi_{\pm}(x_1) \right) \delta_{12}^4 \left[\frac{1}{\square(\square - m_{\tilde{\gamma}}^2)} \right]_1 \delta_{12}^4 \right\} . \tag{49}
\end{aligned}$$

This expression can be evaluated further using the same Fourier transforms and Wick rotations as employed for diagram $\Gamma_A(m_{\tilde{\gamma}}=0)$, and find

$$\begin{aligned}
\Gamma_{\text{full}} &= -e^2 \int \frac{d^4p}{(4\pi)^4} \left\{ \left(\bar{F}_{\pm}(p)F_{\pm}(-p) - \bar{\varphi}_{\pm}(p)p^2\varphi_{\pm}(-p) + i\bar{\Psi}_D(p)\not{p}\Psi_D(-p) \right) L(p^2, m^2) \right. \\
&\quad + \tilde{m}_{\tilde{\gamma}}^2 \left(2\bar{F}_{\pm}(p)F_{\pm}(-p) - 2\bar{\varphi}_{\pm}(p)p^2\varphi_{\pm}(-p) - i5\bar{\Psi}_D(p)\not{p}\Psi_D(-p) \right) L(p^2, \tilde{M}^2) \\
&\quad + m_{\tilde{\gamma}}^2 \left(2\bar{\varphi}_{\pm}(p)p^m p^n \varphi_{\pm}(-p) + i\bar{\psi}_D(p)\gamma^m p^n \psi_D(-p) + n \leftrightarrow m \right) L_{nm}(p^2, \tilde{\mathcal{M}}^2) \\
&\quad \left. - 2m_{\tilde{\gamma}} \left(\bar{F}_{\pm}(p)\varphi_{\pm}(-p) + \bar{\varphi}_{\pm}(p)F_{\pm}(-p) - 2m_{\tilde{\gamma}}\bar{\varphi}_{\pm}(p)\varphi_{\pm}(-p) \right) I(m_{\tilde{\gamma}}^2) \right\} . \tag{50}
\end{aligned}$$

The integrals $I(m_{\tilde{\gamma}}^2)$, and $L(p^2, m^2)$ are divergent, whereas $L(p^2, \tilde{M}^2)$ and $L_{nm}(p^2, \tilde{\mathcal{M}}^2)$ are finite. We have evaluated these integrals in appendix B, (see (B.12), (B.24), (B.24) and (B.25)) respectively, with

$$\tilde{m}^2 = x(1-x)p^2 + xm^2, \quad \tilde{M}^2 = y(1-y)p^2 + m_{\tilde{\gamma}}^2(1-x-y) + m^2y, \tag{51}$$

$$\tilde{\mathcal{M}}^2 = m^2y + m_{\tilde{\gamma}}^2(1-x-y). \tag{52}$$

From this result we can determine the renormalized quantities and a number of additional finite terms that are second order in the coupling constant e . The wave function renormalization Z_1 is obtained from the integral $L(p^2, m^2)$:

$$L(p^2, m^2) = \mu^{-2\epsilon} \int_0^1 dx J_2(m^2) = \frac{\mu^{-2\epsilon}}{16\pi^2} \left[\frac{1}{\epsilon} + 2 - \ln \frac{m^2}{\mu^2} - \frac{p^2 + m^2}{p^2} \ln \frac{p^2 + m^2}{m^2} \right], \tag{53}$$

from which we read off the wave function renormalization Z_1

$$Z_1 = \mu^{-2\epsilon} \left(1 - \frac{e^2}{16\pi^2} \frac{1}{\bar{\epsilon}} \right), \quad (54)$$

that is consistent with (42). Because of the non-renormalization theorem, we also find that the electron mass m did not received quantum correction except for the wave function renormalization: $m = Z_1^{-1} m_R$. The second and the third line of (50) are additional finite renormalization for the photino mass.

5 Counterterms

As a first step towards the renormalization we need to introduce counterterms. From the first example in subsection 4.1, the infinities arise in the first term of (35)

$$i\Gamma_A(m_{\tilde{\gamma}} = 0) = -e^2 \int (d^4x)_{12} d^4\theta \bar{\Phi}(x_2, \bar{\theta}) \Phi(x_1, \theta) \left(\frac{\square - m^2}{(\square - m^2)^2 - M^4} \right) \delta_{12}^4 \frac{1}{\square_1} \delta_{12}^4, \quad (55)$$

which after regularization becomes

$$\Gamma_A(m_{\tilde{\gamma}} = 0) = -\frac{e^2}{2} \int \frac{d^4p}{(4\pi)^4} \int_0^1 dx \left[J_2(\tilde{m}_-^2) + J_2(\tilde{m}_+^2) \right] \int d^4\theta \bar{\Phi}_{\pm}(p, \bar{\theta}) \Phi_{\pm}(-p, \theta). \quad (56)$$

The divergent part of the above expression reads

$$\Gamma_A^{\text{div}}(m_{\tilde{\gamma}} = 0) = -\frac{e^2 \mu^{-2\epsilon}}{16\pi^2} \frac{1}{\bar{\epsilon}} \int \frac{d^4p}{(4\pi)^4} \int d^4\theta \bar{\Phi}_{\pm}(p, \bar{\theta}) \Phi_{\pm}(-p, \theta). \quad (57)$$

In the second example in subsection 4.2 we find that the divergent part take the same form as (57)

$$\Gamma_A^{\text{div}}(m_{\tilde{\gamma}} = 0) = \Gamma_A^{\text{div}}(M = 0). \quad (58)$$

Finally, from the diagram Γ_B give rise to infinite terms that involve spurion superfields θ^2 and $\bar{\theta}^2$:

$$\Gamma_B^{\text{div}} = 2m_{\tilde{\gamma}} \frac{e^2 \mu^{-2\epsilon}}{16\pi^2} \frac{1}{\bar{\epsilon}} \int \frac{d^4p}{(4\pi)^4} \int d^4\theta \left\{ \left(\theta^2 + \bar{\theta}^2 - 2m_{\tilde{\gamma}} \theta^2 \bar{\theta}^2 \right) \bar{\Phi}_{\pm}(p, \bar{\theta}) \Phi_{\pm}(-p, \theta) \right\}. \quad (59)$$

To cancel these infinities (57) and (59) we have to introduce two one-loop counter terms

$$\Delta S = \Delta S_1 + \Delta S_2. \quad (60)$$

The first term cancels the divergent part (57) of the electron self-energy diagram Γ_A and takes the form

$$\Delta S_1 = \Delta Z_1 \int d^8z \left(\bar{\Phi}_+ \Phi_+ + \bar{\Phi}_- \Phi_- \right), \quad (61)$$

where $\Delta Z_1 = Z_1 - 1$. The second counterterm is equal to

$$\Delta S_2 = 2m_{\tilde{\gamma}} \Delta Z_1 \int d^8z \left(-\theta^2 - \bar{\theta}^2 + 2m_{\tilde{\gamma}} \theta^2 \bar{\theta}^2 \right) \left(\bar{\Phi}_+ \Phi_+ + \bar{\Phi}_- \Phi_- \right). \quad (62)$$

Addition of the this term cancels the infinite part (59) of diagram Γ_B .

The fact that loop correction generates divergencies that contains spurion superfields is not surprising. This issue has been discussed before in the literature [4, 8, 9]. Following Yamada [9] these divergencies can be removed by spurion dependent transformations

$$\Phi'_\pm = \Phi_\pm - 2m_{\tilde{\gamma}}\theta^2\Phi_\pm, \quad \overline{\Phi}'_\pm = \overline{\Phi}_\pm - 2m_{\tilde{\gamma}}\bar{\theta}^2\overline{\Phi}_\pm. \quad (63)$$

In summary, we have shown from these examples that there is only one counterterm needed and it is given by

$$\Delta S' = \Delta Z_1 \int d^8z \left(\overline{\Phi}'_+ \Phi'_+ + \overline{\Phi}'_- \Phi'_- \right). \quad (64)$$

6 Summary

Supergraphs is a powerful, elegant and efficient tool for computations even when supersymmetry is softly broken. In these cases supersymmetry breaking are represented by spurions.

In this paper we have represented supersymmetry breaking by spurion superfields, and worked out explicit expressions for the propagators. By using them we calculated exact expressions for the one-loop diagrams that contributes to electron and selectron self energies. In particular, we calculate the one-loop gauge superfield contribution to the two-point function from which one can extract the wave function renormalization for the electron and its superpartner the selectron. Our computation confirm that the divergent part of the wave function renormalization is independent of the soft breaking, and is the same for selectron and the electron. Furthermore, we found there are divergencies that involve spurion superfields which can be removed by Yamada spurion dependent transformations.

Acknowledgments

We would like to thank Stefan Groot Nibbelink for numerous discussions, encouragement, reading the manuscript. We would also like to thank Jan-Willem van Holten for suggesting many useful remarks.

A Identities

In this appendix we give the relations which we have used to reduce the supergraph integral (45) to scalar integral (46). By integrating over the full double superspace, we find the following relations

$$\begin{aligned} \int d^8 z_{12} \delta_{12} [\Delta_V]_1 \delta_{12} &= 2 \int d^4 x_{12} d^4 \theta \delta_{12}^4 \left[\frac{m_{\tilde{\gamma}}}{\square (\square - m_{\tilde{\gamma}}^2)} (\theta^2 + \bar{\theta}^2 - 2 \theta^2 \bar{\theta}^2 m_{\tilde{\gamma}} \right. \\ &\quad \left. + i \bar{\theta}^{\dot{\alpha}} \theta^{\alpha} \sigma_{\dot{\alpha}\alpha}^m \frac{\partial_m}{\square} m_{\tilde{\gamma}}) \right]_1 \delta_{21}^4, \end{aligned} \quad (\text{A.1})$$

$$\int d^8 z_{12} \delta_{12} [\bar{D}^2 D^2 \Delta_V]_1 \delta_{12} = 16 \int d^4 x_{12} d^4 \theta \delta_{12}^4 \frac{1}{\square_1} \delta_{21}^4, \quad (\text{A.2})$$

$$\begin{aligned} \int d^8 z_{12} \delta_{12} [\bar{D}^{\dot{\beta}} \Delta_V]_1 \delta_{12} &= -2 \int d^4 x_{12} d^4 \theta \delta_{12}^4 \left[\left(2 \theta^2 \bar{\theta}^{\dot{\beta}} - i \epsilon^{\dot{\beta}\alpha} \theta^{\alpha} \sigma_{\dot{\alpha}\alpha}^m \frac{\partial_m}{\square} \right) \frac{m_{\tilde{\gamma}}^2}{\square (\square - m_{\tilde{\gamma}}^2)} \right. \\ &\quad \left. - \frac{m_{\tilde{\gamma}}}{\square (\square - m_{\tilde{\gamma}}^2)} \bar{\theta}^{\dot{\beta}} \right]_1 \delta_{12}, \end{aligned} \quad (\text{A.3})$$

$$\int d^8 z_{12} \delta_{12} [\bar{D}^2 D^{\beta} \Delta_V]_1 \delta_{12} = 8 \int d^4 x_{12} d^4 \theta \delta_{12}^4 \left[\left(\theta^{\beta} m_{\tilde{\gamma}} - i \bar{\theta}_{\dot{\beta}} \bar{\sigma}^{m\dot{\beta}\beta} \partial_m \right) \frac{m_{\tilde{\gamma}}}{\square (\square - m_{\tilde{\gamma}}^2)} \right]_1 \delta_{12}, \quad (\text{A.4})$$

$$\int d^8 z_{12} \delta_{12} [\bar{D}^2 \Delta_V]_1 \delta_{12} = 0, \quad (\text{A.5})$$

$$\begin{aligned} \int d^8 z_{12} \delta_{12} [\bar{D}^{\dot{\beta}} D^{\beta} \Delta_V]_1 \delta_{12} &= -2 \int d^4 x_{12} d^4 \theta \delta_{12}^4 \left[\left((i \bar{\theta}^2 \theta^2 \bar{\sigma}^{m\dot{\beta}\beta} \partial_m - 2 \bar{\theta}^{\dot{\beta}} \theta^{\beta}) m_{\tilde{\gamma}} + i (\bar{\theta}^2 + \theta^2) \bar{\sigma}^{m\dot{\beta}\beta} \partial_m \right. \right. \\ &\quad \left. \left. - 2 \bar{\theta}^{\dot{\alpha}} \theta^{\gamma} \epsilon^{\beta\alpha} \epsilon^{\dot{\beta}\dot{\gamma}} \sigma_{\dot{\alpha}\alpha}^m \sigma_{\dot{\gamma}\gamma}^n \frac{\partial_m \partial_n}{\square} m_{\tilde{\gamma}} \right) \frac{m_{\tilde{\gamma}}}{\square (\square - m_{\tilde{\gamma}}^2)} \right]_1 \delta_{12}, \end{aligned} \quad (\text{A.6})$$

$$\begin{aligned} \int d^8 z_{12} \delta_{12} [D^{\beta} \Delta_V]_1 \delta_{12} &= 2 \int d^4 x_{12} d^4 \theta \delta_{12}^4 \left[\left(2 \bar{\theta}^2 \theta^{\beta} - i \epsilon^{\beta\alpha} \bar{\theta}^{\dot{\alpha}} \sigma_{\dot{\alpha}\alpha}^m \frac{\partial_m}{\square} \right) \frac{m_{\tilde{\gamma}}^2}{\square (\square - m_{\tilde{\gamma}}^2)} \right. \\ &\quad \left. + \frac{m_{\tilde{\gamma}}}{\square (\square - m_{\tilde{\gamma}}^2)} \theta^{\beta} \right]_1 \delta_{12}. \end{aligned} \quad (\text{A.7})$$

B One Loop Scalar Integrals

This appendix is devoted to the evaluation of the basic one loop scalar integrals, which arise in the main text of this paper. We compute these scalar integrals in the $\overline{\text{MS}}$ scheme: We evaluate the integrals in $D = 4 - 2\epsilon$ dimensions, and we introduce the renormalization scale μ such that all D dimensional integrals have the same mass dimensions as their divergent four dimensional counter parts.

The three basic type one loop integrals are given by

$$J_\alpha(M^2) = \int \frac{d^D p}{(2\pi)^D \mu^{D-4}} \frac{1}{(p^2 + M^2)^\alpha} = \frac{1}{16\pi^2} \frac{1}{(M^2)^{\alpha-2}} \left(4\pi \frac{\mu^2}{M^2}\right)^{2-\frac{D}{2}} \frac{\Gamma(\alpha - \frac{D}{2})}{\Gamma(\alpha)}, \quad (\text{B.8})$$

$$T_{\alpha mn}(M^2) = \int \frac{d^D p}{(2\pi)^D \mu^{D-4}} \frac{p_m p_n}{(p^2 + M^2)^\alpha} = \frac{1}{D} \eta_{mn} \left(J_{\alpha-1}(M^2) - M^2 J_\alpha(M^2) \right), \quad (\text{B.9})$$

$$I(M^2) = \int \frac{d^D p}{(2\pi)^D \mu^{D-4}} \frac{1}{p^2} \frac{1}{p^2 + M^2} = \frac{1}{16\pi^2} \left(4\pi \frac{\mu^2}{M^2}\right)^{2-\frac{D}{2}} \frac{1}{\Gamma(\frac{D}{2})} \frac{\pi}{\sin \pi(\frac{D}{2} - 1)} \quad (\text{B.10})$$

for $\alpha = 2, 3, 4$. In the applications in the main text we need to expand this to the zeroth order in ϵ including the pole $1/\epsilon$:

$$J_2(M^2) = \frac{1}{16\pi^2} \left[\frac{1}{\epsilon} - \ln \frac{M^2}{\bar{\mu}^2} \right], \quad J_3(M^2) = \frac{1}{32\pi^2} \frac{1}{M^2}, \quad (\text{B.11})$$

$$I(M^2) = \frac{1}{16\pi^2} \left[\frac{1}{\epsilon} + 1 - \ln \frac{M^2}{\bar{\mu}^2} \right], \quad J_4(M^2) = \frac{1}{96\pi^2} \frac{1}{M^4}. \quad (\text{B.12})$$

Here we have introduced the $\overline{\text{MS}}$ scale $\bar{\mu}^2 = 4\pi e^{-\gamma_E} \mu^2$ with Euler constant γ_E .

The second three integrals we encounter in this work are

$$L(p^2, M^2) = \int \frac{d^D q}{(2\pi)^D \mu^{D-4}} \frac{1}{(q + p/2)^2 + M^2} \frac{1}{(q - p/2)^2}, \quad (\text{B.13})$$

$$L(p^2, M_1^2, M_2^2) = \int \frac{d^D q}{(2\pi)^D \mu^{D-4}} \frac{1}{(q + p/2)^2 + M_1^2} \frac{1}{(q - p/2)^2 + M_2^2} \frac{1}{(q - p/2)^2}, \quad (\text{B.14})$$

$$L_{mn}(p^2, M_1^2, M_2^2) = \int \frac{d^D q}{(2\pi)^D \mu^{D-4}} \frac{q_m q_n}{[(q + p/2)^2 + M_1^2][(q - p/2)^2 + M_2^2][(q - p/2)^2]^4} \quad (\text{B.15})$$

Using Feynman parametrizations

$$\frac{1}{AB} = \int_0^1 dx \frac{1}{[xA + (1-x)B]^2}, \quad (\text{B.16})$$

$$\frac{1}{ABC} = 2 \int_0^1 dx \int_0^{1-x} dy \frac{1}{[xA + yB + (1-x-y)C]^3} \quad (\text{B.17})$$

$$\frac{1}{A^2 BC} = 6 \int_0^1 dx \int_0^{1-x} dy \frac{x}{[xA + yB + (1-x-y)C]^4}, \quad (\text{B.18})$$

we rewrite the $L(p^2, M^2)$, $L(p^2, M_1^2, M_2^2)$ and $S_{mn}(M_1^2, M_2^2)$ in a more convenient form, namely

$$L(\tilde{m}^2) = \frac{1}{2} \int_0^1 dx \int \frac{d^D \tilde{q}}{(2\pi)^D \mu^{D-4}} \frac{1}{(\tilde{q}^2 + \tilde{m}^2)^2} \quad (\text{B.19})$$

$$L(\tilde{M}^2) = 2 \int_0^1 dx \int_0^{1-x} dy \int \frac{d^D q'}{(2\pi)^D \mu^{D-4}} \frac{1}{(q'^2 + \tilde{M}^2)^3} \quad (\text{B.20})$$

$$L_{mn}(p, \tilde{\mathcal{M}}^2) = 6 \int_0^1 dx \int_0^{1-x} dy \int \frac{d^D q'}{(2\pi)^D \mu^{D-4}} \left[x \frac{q'_m q'_n}{(q'^2 + \tilde{\mathcal{M}}^2)^4} + x y^2 \frac{p_m p_n}{(q'^2 + \tilde{\mathcal{M}}^2)^4} \right]. \quad (\text{B.21})$$

where

$$\begin{aligned} \tilde{m}^2 &= x(1-x)p^2 + xM^2, & \tilde{M}^2 &= y(1-y)p^2 + M_1^2 y + M_2^2(1-x-y) \\ \tilde{\mathcal{M}}^2 &= yM_1^2 + (1-x-y)M_2^2. \end{aligned} \quad (\text{B.22})$$

In obtaining these formula we have shifted the integration variables $\tilde{q} = q + (x - \frac{1}{2})p^2$ and $q' = q - \frac{1}{2}(1-2v)p$. It is not difficult to confirm that these integrals can be written in terms of the simple integrals J_2 and J_3 as

$$\begin{aligned} L(p^2, \tilde{m}^2) &= \int_0^1 dx J_2(\tilde{m}^2) = \frac{1}{16\pi^2} \left[\frac{1}{\epsilon} + 2 - \ln \frac{M^2}{\mu^2} - \frac{p^2 + M^2}{p^2} \ln \frac{p^2 + M^2}{M^2} \right] \\ &= \frac{1}{16\pi^2} \left[\frac{1}{\epsilon} - \gamma + \ln(4\pi) + 2 - \ln \frac{M^2}{\mu^2} - \frac{p^2 + M^2}{p^2} \ln \frac{p^2 + M^2}{M^2} \right] \end{aligned} \quad (\text{B.23})$$

$$L(p^2, \tilde{M}^2) = 2 \int_0^1 dx \int_0^{1-x} dy J_3(\tilde{M}^2) = \frac{1}{16\pi^2} \int_0^1 dx \int_0^{1-x} dy \frac{1}{\tilde{M}^2} \quad (\text{B.24})$$

$$\begin{aligned} L_{mn}(p^2, \tilde{\mathcal{M}}^2) &= 6 \int_0^1 dx \int_0^{1-x} dy \left[x \left\{ T_{4mn}(\tilde{\mathcal{M}}^2) + y^2 p_m p_n J_4(\tilde{\mathcal{M}}^2) \right\} \right] \\ &= \frac{1}{2\pi^2} \int_0^1 dx \int_0^{1-x} dy \left[\frac{x}{\tilde{\mathcal{M}}^2} \left\{ \frac{y^2 p_m p_n}{8\tilde{\mathcal{M}}^2} - \eta_{mn} \right\} \right] \end{aligned} \quad (\text{B.25})$$

References

- [1] P. Fayet and J. Iliopoulos “Spontaneously Broken Supergauge Symmetries and Goldstone Spinors” *Phys. Lett.* **B51** (1974) 461–464.

- [2] O. Piguet, K. Sibold, and M. Schweda “RADIATIVE MASS GENERATION IN PERTURBATION THEORY AND THE RENORMALIZATION GROUP” *Nucl. Phys.* **B168** (1980) 337.
- [3] K. Sibold “RENORMALIZATION OF EXTENDED QED WITH SPONTANEOUSLY BROKEN SUPERSYMMETRY” *Nucl. Phys.* **B174** (1980) 491.
- [4] L. Girardello and M. T. Grisaru “Soft breaking of supersymmetry” *Nucl. Phys.* **B194** (1982) 65.
- [5] S. G. Nibbelink and T. S. Nyawelo “Effective action of softly broken supersymmetric theories” *Phys. Rev.* **D75** (2007) 045002 [[hep-th/0612092](#)].
- [6] S. J. Gates, M. T. Grisaru, M. Rocek, and W. Siegel “Superspace, or one thousand and one lessons in supersymmetry” *Front. Phys.* **58** (1983) 1–548 [[hep-th/0108200](#)].
- [7] P. C. West “Introduction to supersymmetry and supergravity”. Singapore, Singapore: World Scientific (1990) 425 p.
- [8] D. Chang and A. Gangopadhyaya “Renormalization group equations in broken SUSY theories using superspace methods” *Phys. Rev.* **D33** (1986) 1771.
- [9] Y. Yamada “Two loop renormalization group equations for soft susy breaking scalar interactions: Supergraph method” *Phys. Rev.* **D50** (1994) 3537–3545 [[hep-ph/9401241](#)].
- [10] J. Wess and J. Bagger “Supersymmetry and supergravity”. Princeton, USA: Univ. Pr. (1992) 259 p.
- [11] B. A. Ovrut and J. Wess “Supersymmetric R(xi) gauge and radiative symmetry breaking” *Phys. Rev.* **D25** (1982) 409.
- [12] S. Groot Nibbelink and T. S. Nyawelo “Two loop effective Kähler potential of (non-)renormalizable supersymmetric models” *JHEP* **01** (2006) 034 [[hep-th/0511004](#)].
- [13] J. A. Helayel-Neto “Superpropagators for explicitly broken supersymmetric theories” *Phys. Lett.* **B135** (1984) 78–84.
- [14] W. Siegel “Supersymmetric dimensional regularization via dimensional reduction” *Phys. Lett.* **B84** (1979) 193.
- [15] D. M. Capper, D. R. T. Jones, and P. van Nieuwenhuizen “Regularization by dimensional reduction of supersymmetric and nonsupersymmetric gauge theories” *Nucl. Phys.* **B167** (1980) 479.
- [16] B. De Wit and J. Smith “FIELD THEORY IN PARTICLE PHYSICS. VOL. 1”. Amsterdam, Netherlands: North-holland (1986) 490 P. (Personal Library).
- [17] K. Inoue, A. Kakuto, H. Komatsu, and S. Takeshita “Aspects of grand unified models with softly broken supersymmetry” *Prog. Theor. Phys.* **68** (1982) 927.
- [18] K. Inoue, A. Kakuto, H. Komatsu, and S. Takeshita “Renormalization of supersymmetry breaking parameters revisited” *Prog. Theor. Phys.* **71** (1984) 413.
- [19] J. P. Derendinger and C. A. Savoy “Quantum effects and SU(2) x U(1) breaking in supergravity gauge theories” *Nucl. Phys.* **B237** (1984) 307.
- [20] B. Gato, J. Leon, J. Perez-Mercader, and M. Quiros “Renormalization group analysis for a general softly broken supersymmetric gauge theory” *Nucl. Phys.* **B253** (1985) 285.
- [21] N. K. Falck “Renormalization group equations for softly broken supersymmetry: The most general case” *Z. Phys.* **C30** (1986) 247.
- [22] S. P. Martin and M. T. Vaughn “Two loop renormalization group equations for soft supersymmetry breaking couplings” *Phys. Rev.* **D50** (1994) 2282 [[hep-ph/9311340](#)].

- [23] M. T. Grisaru, W. Siegel, and M. Rocek “Improved methods for supergraphs” *Nucl. Phys.* **B159** (1979) 429.
- [24] S. Weinberg “Non-renormalization theorems in non-renormalizable theories” *Phys. Rev. Lett.* **80** (1998) 3702–3705 [[hep-th/9803099](#)].
- [25] E. Poppitz and L. Randall “Holomorphic anomalies and the nonrenormalization theorem” *Phys. Lett.* **B389** (1996) 280–286 [[hep-th/9608157](#)].
- [26] N. Seiberg “Naturalness versus supersymmetric nonrenormalization theorems” *Phys. Lett.* **B318** (1993) 469–475 [[hep-ph/9309335](#)].
- [27] I. L. Buchbinder and S. M. Kuzenko “Ideas and methods of supersymmetry and supergravity: Or a walk through superspace”. Bristol, UK: IOP (1998) 656 p.